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MATH 4010

Fourier, Jean Baptiste Joseph

The French mathematical physicist Jean Baptiste Joseph Fourier (1768-1830) was the first to discuss, in a comprehensive manner, the various aspects of the flow of heat in bodies. His accomplishments in the mathematical and physical world have left an immeasurable impact on our own civilization and his methods are indispensable today.

Jean Baptiste Joseph Fourier was born southeast of Paris in Auxerre, France on March 21, 1768. Jean Baptiste was the son of a tailor, Joseph Fourier, whose wife was Edmée Fourier. By the age of 8, Joseph's father had died, and less than a year after this tragedy his mother had passed away, leaving Joseph Fourier an orphan by the age of 8[1]. Shortly after, a charitable lady who had been captivated by the boy's good manners and serious deportment recommended him to the Bishop of Auxerre. The Bishop got Fourier into the local military college run by Benedictine Monks, where the boy soon proved his genius. By the age of twelve Fourier was writing magnificent sermons for the leading church dignitaries of Paris to palm off as their own. At thirteen he was a problem child, disobedient, ill-tempered, and full of the devil generally. Then, at his first encounter with mathematics, he changed as if he had been reborn by magic. He figured out what had been making him such a problem child and cured himself of it.[5] Fourier became so obsessed with mathematics that after his bedtime he would collect candle-ends from the kitchen and anywhere else he could find them in the college to provide light for his mathematical studies.[5] He had a secret study in a nook by a large fireplace behind a screen.

While staying at the local military school Fourier wanted to join either the artillery or the engineers, which were branches of the military then generally available to all classes of society; but Fourier was turned down for some reason. [12] The good Benedictine monks prevailed upon the young genius to choose the priesthood as his profession, and he entered the abbey of St. Benoit-sur-Loire to become a novitiate. He had always wanted to be a solider and had chosen the priesthood only because commissions were not given to sons of tailors. [9] The French Revolution interfered with these plans and set him free. [5] His old friends at Auxerre knew all a long that Fourier would never make a monk so they took him back in 1789 and made him a professor of mathematics at his old school. This was the first step (indeed a long one) toward his ambition. Fourier proved his versatility by teaching his colleagues' classes when they were ill, usually better than they did themselves, in everything from physics to the classics.

In December 1789, Fourier (21 years of age) went to Paris to present his researches on the solution of numerical equations before the Academy. [5] This work was beyond Lagrange and is still of value. Fourier's first memoir on the numerical solution of algebraic equations was read before the French Academy of Sciences. The subject became one of his life-long interests. [5]

Fourier, of course, lived during the French Revolution and was an enthusiast for the Revolution from the start. [6] Then the Revolution got out of hand and Fourier protested against the needless brutality. [6] He was prominent in local affairs and his courageous defense of the victims of the Terror led to his arrest in 1794. He personally appealed to Robespierre, who led the Reign of Terror, which turned out to be unsuccessful. Fourier was later released when Robespierre was executed on July 28, 1794.

It is to Napoleon's everlasting credit that he was one of the first to see with cold-blooded clarity that ignorance of itself can do nothing but destroy.[5] His own work may not have been much better in the end, but he did recognize that such a thing as civilization might be possible. Then Napoleon ordered and encouraged the creation of schools. The problem that arose was the lack of teachers at that time. The École Normale was created in 1794 because it was imperative to train new teaching corporations of over fifteen hundred. Fourier was called to the chair of mathematics as a reward for his recruiting in Auxerre. With his new appointment began a new era in teaching French mathematics. Fourier remembered the deadly lectures of redundant professors who memorized and delivered the same lectures year after year, the Convention called in creators of mathematics to do the teachings. He forbade them to lecture from any notes at all. The lectures were to be given standing and not sitting half asleep behind a desk. The professors and the class were free to interchange questions and explanations, but it was up to the professor to prevent lecture from turning into a pointless debate. The École Normale was opened and closed within a year and in that case Fourier only spent a short time there but nevertheless he made a strong impression. The success of his scheme surpassed expectations and led to one of the most brilliant periods in the history of French mathematics and science.

Shortly after in 1795 the École Polytechnique started and he was appointed *administrateur de police*, or assistant lecturer, to support the teaching of Lagrange and Monge then later as a professor of mathematical analysis. At Polytechnique he enlivened his lectures on mathematics by out-of-the-way historical allusions (many of which he was the first to trace to their sources), and he skillfully tempered abstractions with interesting applications. [5] There, at Polytechnique, Fourier became victim of the reaction to the previous regime and was, ironically, arrested as a supporter of Robespierre. Robespierre had declined his earlier appeal when Fourier was first arrested. His colleagues at École successfully sought his release. [12]

In 1798, Monge selected Fourier to join Napoleon's Egyptian campaign as one of the Legion of Culture to civilize Egypt—"to offer a succoring hand to unhappy peoples, to free them from the brutalizing yoke under which they have groaned for centuries, and finally to endow them without delay with all the benefits of European civilization." Fourier became a secretary of the newly formed Institute d'Egypte, where he became Napoleon's right hand man who conducted negotiations between Napoleon and Sitty-Neficah (the wife of the chief bey, Murad). Fourier also held other diplomatic posts as well as pursuing research in Egypt. [5] He was 30 when Napoleon requested his participation as scientific adviser on an expedition to Egypt. From 1798 to 1802 Fourier served as secretary of the Institut d'Egypte, which was established by Napoleon to explore the archeological riches of that ancient land. In his papers published in the *Decade* and the *Courrier d'Egypte* it showed him to be preoccupied with problems that ranged from the general solutions of algebraic equations to irrigation projects.

On January 2, 1802 Fourier had finally retuned to France and was appointed prefect of the Department of Isere, with headquarters at Grenoble. [13] Grenoble extended to what was then the Italian border. His first task was to restore order there because it was in political turmoil. The citizens of Grenoble were much upset by the religious implications of some of the Institute d'Egpyte's discoveries, particularly the great age assigned to the older monuments, which conflicted with the chronology of the Bible. They calmed down when Fourier dug up a saint in his own family, the blessed Pierre Fourier, his great-uncle, whose memory was hallowed because he had founded a religious order. The archeological researches nearer home to the Grenoble citizens made them quite satisfied with Fourier. Now with his respectability regained, Fourier accomplished a vast amount of useful work such as reconciliation of thirty-seven different communities to the drainage of a huge area of marshland near Bourgoin into valuable land, stamping out malaria, planning and partial construction of a road from Grenoble to Turin, and overall lifting his district out of the Middle ages. [5]

While at the Department of Isere, he wrote the work on the mathematical theory of heat conduction which had earned him lasting fame. His first draft was submitted to the academy in 1807 and a second more detailed version was sent which received an award from the academy.[8] His paper was entitled *Theorie des mouvements de la chleur dans les corps solides*. The first part of his work was printed in book form in 1822 under the title *Theorie analytique de la Chaluer* (The Mathematical Theory of Heat).[8] His work became a masterpiece because it covered the hitherto unexplored field of heat propagation but also because it contained the mathematical techniques which later were developed into a special branch of mathematics—Fourier analysis and Fourier integrals. The Academy encouraged Fourier to continue by setting a contribution to the mathematical theory of heat as its problem for the Grand Prize in 1812. Fourier won the prize, but not without some criticism which he resented deeply but which was well taken.

Laplace, Lagrange, and Legendre were the referees. While admitting the novelty and importance of Fourier's work they pointed out that the mathematical treatment was faulty. Fourier analysis is the decomposition of a function in the terms of a sum of sinusoidal functions of different frequencies that can be recombined to obtain the original function. [14] The Fourier series is a type of Fourier analysis, which decomposes a periodic function into a sum of simple functions that may contain sines, cosines or complex exponentials[4].

Imagine we have a block of metal and the surface is maintained at a fixed temperature. Then under physical considerations the interior temperature distributions at initial time, t=0, is all you need to find the interior distributions at all future times. [7] Even if the initial temperature distribution changes Fourier asserted that "nevertheless it is equal to the sum of a series of sines and cosines" [4]. Here Fourier was repeating Bernoulli except for the fact that Bernoulli was only speaking of functions that are formed by a single expression and Fourier included those functions given piecewise by several different formulas[3]. Fourier was asserting that the distinction of function and graph was nonexistent. Since every function represents a graph, then every graph represents a function, called its Fourier series[2]. Lagrange, one of the creators of variational calculus, and many other great mathematicians found Fourier's claim very hard to believe[1].

Fourier's most important step was finding the formula of the coefficients in the expansion. Without knowing that Euler and Bernoulli had calculated this previously Fourier calculated it all over again. All three of the mathematicians looked past the direct method of orthogonality, which could have solved the problem in just one line of work[4].

Fourier went through an incredible computation, which has been called a classic example of physical insight leading to the right answer in spite of flagrantly wrong reasoning[2].

Fourier started out expanding each sine function in a power series and then rearranged the terms so that the arbitrary function f was represented by a power series. This was the first sign of wrong reasoning by Fourier because the functions he had in mind have no such expansion in general. This did not stop Fourier as he proceeded to find the coefficients of this nonexistent power-series expansion. Using his inconsistent assumptions he arrived at answer where he used division by an arbitrarily large number. This formula concluded that the arbitrary function is zero because all the coefficients vanish. Unhappy with the results Fourier manipulated his solution to arrive at the same simple formula Euler had concluded over thirty years prior[2].

After the vigorous calculations Fourier accomplished to arrive at Euler's formula, he realized that he could have used the direct method of orthogonality to compute the problem in one line, just as Euler did[2]. Fourier was the first to observe that the final coefficient formula remained meaningful for any graph bounded by a definite area. He then concluded that this worked for any and all graphs. With his previous computations of the Fourier series for a number of special examples, he soon found that numerically in every case that the sum of the first few terms was very close to the actual graph that generated the series. With this in mind, he claimed that every graph, no matter how many pieces it consists of, is representable by a series of sines and cosines[2]. Fourier was correct, but he never stated or proved a correct theorem of the Fourier series. It took nearly a century of work by mathematicians to make sense of what Fourier did to accomplish the Fourier series.

Fully aware that he had done something of the first magnitude Fourier paid no attention to his critics. They were right, he was wrong, but he had done enough in his own way to entitle him to independence. Fourier's resentment was rationalized in attacks on pure mathematicians was rationalized in attacks on pure mathematicians for minding their own proper business and not blundering about in mathematical physics. [11]

On March 1, 1815, Napoleon had escaped from Elba and landed on the French coast. All was going well for Fourier and France in general. [13] Fourier started to fear that the populace would welcome Napoleon back for another spree, Fourier hurried to Lyons to tell the Bourbons what was about to happen. [14] They refused to believe him and Fourier was taken prisoner and brought before Napoleon at Bourgoin. There he was confronted by Napoleon who was bending over a map, a pair of compasses in his hand and said:

"Well, Monsieur Prefect! You too; you have declared war against me?" [5]

"Sire," Fourier stammered, "my oaths made it a duty." [5]

"A duty, do you say? Don't you see that nobody in the country is of your opinion? And don't let yourself imagine that your plan of campaign frightens me much. I suffer only at seeing amongst my adversaries an *Egyptian*, a man who has eaten the bread of the bivouac with me, an old friend! How, moreover, Monsieur Fourier, have you been able to forget that I made you what you are?" [5]

A few days later Napoleon had asked the now loyal Fourier:

"What do you think of my plan?" [5]

"Sire, I believe you will fail. You will meet a fanatic on your road, and everything will be over." [5]

"Bah! Nobody is for the Bourbons—not even a fanatic. As for that, you have read in the papers that they have put me outside the law. I myself will be more indulgent: I shall content myself with putting them outside the Tuileries!" [5]

Before the end of Napoleon's Hundred Days, Fourier had resigned his new title and prefecture in protest against the severity of the regime and had come to Paris to try to take up research full time, which had previously been part-time activity.

This was the low point of Fourier's life. He had no job, only a small pension, and low political reputation. But before he could starve to death a former student at the Ecole Polytechnique and companion in Egypt, Chabrol de Volvic, was now prefect of the department of the Seine and appointed him director of its Bureau of Statistics, which was a position without heavy duties but came with a salary good enough for his needs. [11]

In 1816 Fourier was elected to the reconstituted Academie des Sciences, but Louis XVIII could not forgive him for having accepted the prefecture of the Rhone from Napoleon, and was refused. Later some diplomatic negotiation eventually cleared up the situation and he was elected in 1817. In 1822, he was elected to the powerful position *secretaire perpetual* of the Academic des Sciences, and in 1827 was elected to the Academie Francaise. He was also elected a foreign member of the Royal Society.

Fourier's obsession with heat and his mathematical theory of heat conduction were responsible for hastening his death. Fourier loved to sweat so much that he would go into saunas in layers of clothes. [13] His experiences in Egypt led him to believe that the desert heat was the ideal condition for health. His friends stated that Fourier lived in rooms that were hotter than hell and the Sahara desert combined! On May 16, 1830 Fourier died of heart disease (some say an aneurism). He was sixty-three years old when he passed. During the course of his career Fourier wrote several papers on statistics, but his lifelong love was the theory of algebraic equations on which he had just completed the manuscript of a book, *Analyse des equations determinees*, and a lengthy memoir when he died. Fourier was buried in the eighteenth division of the cemetery of Pere Lachaise. He belongs to a select group of mathematicians whose work is so fundamental that their names have become adjectives in every civilized language.

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