Sophie Germain

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Sophie Germain was a brilliantly passionate intellectual from a very young age. Although she was not interested in mathematics until the age of 13, her love of the art was fostered and strengthened through social resistance. This resistance was the tone of her early life and defined her later life.

Sophie was born on April 1, 1776 in Paris, France. She was the daughter of Ambroise-Francois, a prosperous silk merchant and Marie-Madelaine Gruguelin. She was exposed to political and philosophical dialogue as a child because her house was often used as a meeting place for liberal social reformers. Outside her home, a revolution was amidst and anarchy ran rampant in the streets. This was a perfect opportunity for a girl like Sophie to envelope herself in mathematics. (Osen, 83)

It all started with J.E. Montucia's account of Archimedes' death. The story goes; Archimedes was killed by a soldier while absorbed in a geometry problem. Her interest in geometry grew and she taught herself Latin and Greek in order to study Euler and Newton. She became quite obsessed with her studies to the point of making her parents concerned her.

Her parents tried desperately to make Sophie put down her books. They went so far as depriving her of heat, light, and according to some sources, even clothing during the night to force her into bed for sleep. Sophie, however, stashed quilts and candles and was found curdled up next to her books morning after morning. Her desperation to learn eventually won over her parents concerns for her well-being and they finally allowed her to study to her hearts content. Although she was never given a tutor, and was completely self taught, her father supported her financially throughout her life so that she would not have to marry or take up an occupation.

In 1794, her studies led her to collecting lecture notes from the newly opened Ecole Polytechnique. Although the school did not allow female students, she diligently collected notes from various professors but was especially captivated by the lectures of J.L. Lagrange. At the end of the course, she began corresponding to Lagrange by posing as a M. le Blanc, a student at the Ecole Polyetchnique. Her analysis impressed Lagrange so much that he sought out Sophie and supported her throughout her career.

Germain continued to work in the field of mathematics through corresponding with several dynamic mathematicians. She responded to Legendre's 1798 *Essai sur le Theorie does Nombres* (Essay on Number Theory) with potential problems. Their correspondences were a colaberative work from which Legendre explicated on in a supplement to his second edition of the *Theorie*.

Germain most famously began writing to Gauss after he published his *Disquisitiones arithmeticae* in 1801. She once again used her pseudonym in these letters in order to avoid being overlooked. Gauss' paper was an extraordinary and complex look into theories of cyclotomy and arithmetical forms which captured Sophie interest. She did her own work on number theory and sent her results to Gauss who was intrigued by her investigation. This led to a close academic relationship between the two as they corresponded for years. In 1807, though, trouble arose when Sophie grew worried about her counterpart as the French occupation came near Gauss' hometown of Braunschweig. She contacted a French commander and family friend, General Pernety, to protect Gauss. Although Gauss was perfectly safe, this action led Gauss to question the true identity of his intellectual comrade. Before and after this discovery Gauss gave high praise to

Germain's work, but upon finding out her gender, Gauss grew a great respect and admiration of Sophie's determination. He writes:

> "But how to describe to you my admiration and astonishment at seeing my esteemed correspondent M. LeBlanc metamorphose himself into this illustrious personage [Sophie Germain] who gives such a brilliant example of what I would find it difficult to believe. A taste for the abstract sciences in general and above all the mysteries of numbers is excessively rare; one is astonished at it; the enchanting charms of this sublime science reveal themselves only to those who have the courage to go deeply into it…Indeed nothing could prove to me in so flattering and less equivocal manner that the attractions of this science, which has enriched my life with so many joys, are not chimerical, as the predilection with which you have honored it." (Bell 1937, pg 262)(Osen, 86-87)

Despite Gauss' and Germain's mutual respect and lifelong relationship, the two were never able to meet face to face.

Germain's early career concerned primarily number theory. She "demonstrated the impossibility of solving Fermat's last theorem if x,y, and z are not divisible by an add prome n. If n is an odd prime less than 100, the equation $x^n + y^n = z^n$ is not soluble in integers not divisible by n."(Osen, 91) Through her work, Germain made her major contribution of Germain Primes. In tackling Fermat's last theorem, proving for the first time that the theorem is true for certain prime numbers, now aptly named Germain primes. These primes occur when p and 2p+1 are both prime. The first few examples of Germain primes are 2, 3, 5, 11, 23, 29, 41, 53, 83, 89, 113, 131 and so on. Today, Germain primes continue to be a hot topic among number theorists as they strive to find more and potentially prove their infinite quantity. It is said that Germain's work "was to remain the most important result related to Fermat's last theorem from1738 until the contribution of Kummer in 1840." (www.agnesscott.edu/lriddle/women/germain-FLT/sgandFLT.html)

Sophie's main interest though, revolved around the work of one Ernst Chladni and consumed nearly a decade of her life. Chladni was interested in the underlying mathematical laws in the vibration of elastic surfaces. He noted the figures formed when he stung a bow against the edges of an elastic plate covered in fine powder. This work peaked interest in a few brave and dedicated souls willing to venture in to a two-dimensional analysis. Lagrange himself declared that this problem would yield only to a radically new system of analysis and that the mathematical methods available were inadequate. In 1808 the French Academy posed the fallowing challenge:

> *Formulate a mathematical theory of elastic surfaces and indicate just how it agrees with empirical evidence.*

Since Lagrange was familiar with the problem and Sophie had learned early in her life that seemingly impossible tasks are a worthy cause. She took on the project despite her lack of formal education which was a large handicap in tackling such a large problem. (www-history .mcs.st-andrews.ac.uk/biographies/germain.html)

By 1811, Germain was able to submit her first attempt in a memoir to the Academy. Although her entry was rejected because of her unsatisfactory leap from an analysis of lines to conclusions about surfaces, she persisted. Her next attempt was in 1813 when another competition was held. This entry earned an honorable mention.

Finally, in 1816, as Germain continued her work, she was able to complete her *Memoir on the Vibrations of Elastic Plates* which won her the prize she sought.

Although she did not appear at the awards ceremony in acceptance of her award and she believed that "the judges did not fully appreciate her work" and "the scientific community did not show the respect that seemed due to her", the honor from the Academy nevertheless boosted Sophie's position in the mathematical world. She was now one of the most noted mathematicians in the world. Prestigious circles welcomed her, and she was finally able to attend sessions at the Institute de France. This was the first time a woman was allowed to do such a thing at the Institute. The award signified the peak of Germain's career.
(www-history .mcs.st-andrews.ac.uk/biographies/germain.html)

Although Sophie felt underappreciated, she had a tremendous impact on those around her. M. H. Navier, one of the judges wrote on the paper "it is a work of which few men are able to read and which only one woman was able to write." Several others in the field wrote very well of her and looked highly upon her work despite the mountain of gender prejudice in the scientific world at the time.

Sophie continued to publish papers on the topic of elasticity, of varying significance. One memoir, considered her most important on elastic surfaces, described the nature, bounds and extent of elastic surfaces. Others focused on the principles of analysis she used in finding the solution of the problem she spent 8 years of her life working at as well as an exploration of the curvature of elastic surfaces. One paper was not published until after her death. It defined the mean curvature as the sum of the reciprocals of the radii of the pre principal curvature or in other words, twice the mean curvature. Sophie heavily used Gauss' work in this last paper but was criticized by some for not understanding the full potential of Gauss' ideas.

In her later years Sophie outlined a Philosophical essay, *Considerations generales sur l'etat des sciences et des letters* in *Oeuvres philosophiques* which was published posthumously. This essay was highly praised for its unique perspective. Sophie dabbled in philosophy, chemistry, physics, geography and even history throughout her life. But she is best known for her analytic genius in mathematics. It seems that her biggest contribution though, was not mathematics itself, rather her incredible strength in overcoming social obstacles as a woman working in the sciences. She made great leaps by use of her brilliance and strength in following her passion.

In 1829, breast cancer became Germain's new biggest challenge. She continued her studies until her death at age 55. By Gauss' request, Sophie was to be given an honorary PhD from the University of Gottingen but died in before the degree could be awarded. One Biographer beautifully describes how even post-life Germain remained a victim of her time:

> "All things considered, she was probably the most profoundly intellectual woman that France has ever produced. And yet, strange as it may seem, when the state official came to make out the death certificate of this eminent associate and co-worker of the most illustrious member of the French Academy of Sciences, he designated her as a rentiere-annuitant-not as a mathemeticienne. Nor is this all. When the Eiffel Tower was erected, in which the engineers were obliged to give special attention to the elasticity of the materials used, there were inscribed on this lofty structure the names of seventy-two servants. But one will not find in this list

> the name of that daughter of genius, whose researches contributed so much toward establishing the theory of the elasticity of metals-Sophie Germain. Was she excluded form this list for the same reason that Agnesi was ineligible to membership in the French Academy-because she was a woman? It would seem so. If such, indeed, was the case, more is the shame for those who were responsible for such ingratitude toward one who had deserved so well of science and who by her achievements had sown on enviable place in the hall of fame." (Mozans, 1913 p. 156)(Osen, 91-93)

Sophie Germain is unanimously known for her strength in defying what society expected from her. She blew academicians out of the water and did it all of her own accord, running only under the support of a few. In a time of revolution, Sophie fought the good fight in the best way. She challenged the societal view of women while devoting herself to discovering inner intricacies of the mathematical world. No violence, just brilliance.

Bibliography

Katz, Victor A History of Mathematics: An Introduction, 2nd Ed., Addison-Wesley, 1998

Osen, Lynn M. Women in Mathematics. Cambridge: the Massachusetts Institute of Technology, 1974

http://www-history .mcs.st-andrews.ac.uk/biographies/germain.html; March 25th, 2008

http://www.agnesscott.edu/lriddle/women/germain-FLT/sgandFLT.html; March 25th, 2008