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Biography Paper: Francois Viete

"Nullum non problema solvere"

Francois Viete (1540-1603) is often considered to be the father of algebra as we see it today; his revolutionary look at how equations are used to solve problems changed the methods behind analysis and allowed for the progression to algebra we see today. Viete's way of implicating new notation and theory behind equations allowed for the ancient Greek method to be completely rebuilt, opening the doors for others to investigate problems in a whole new way. The ideology behind analysis and synthesis, as described by Pappus before him, was the foundation behind his introduction of new techniques for looking at equations and their application. His notation proved to be a large step in the direction of the symbolic analysis that we find in textbooks today, allowing for a more organized method of problem solving. The mathematicians following him, particularly Fermat and Descartes, undoubtedly built off his technique, which effectively allowed algebra to emerge into new areas of math, breaking the constraints of traditional geometric constructions and rudimentary analytic methods. Even though through his life mathematics was seen simply as an avocation, his work permitted countless progressions that were described through the revolutionary work, *The Analytic Art.*

In order to find the setting for Viete's genius, his personal life must be examined. His techniques and revolutionary techniques are characteristic of the "new" science that identified the Renaissance in the sixteenth century [1]. Changes and progressions in all areas of knowledge were taking place, and Viete played a major role in changing the face of mathematics, as he built off the ancient constructions and analysis of those before him.

Viete was born in 1540 in Fontenay-le-Conmte, which is located in what is known as the Vendée today, in the province of Poitou [1]. He was the son of Entienne Viete, a lawyer and first cousin of Barnabe Brisson, who was a former president of the Parliment de Paris. Due to his family's somewhat high standing, and their Catholic nature, he studied at the local Franciscan cloister until he entered the University of Poitiers at the age of 18 to study his father's profession. Upon return to Fontennay in 1559, he began a successful law practice where he served many notable clients that allowed him to build a good reputation. Examples of his clients include Mary Stuart and Queen Eleanor of Austria, and with his growing reputation, he gained the title of Sieur de la Bigotiere. By taking over the legal duties foe the Soubise family in 1564, he became the private secretary to Antoinette d'Aubeterre, who married Jean de Parthenay-l'Archeveque, who needed aid in dealing with recriminations against him from when the Catholics besieged Lyon in 1553 [1]. In addition to his services as a lawyer, he tutored their daughter Catherine, an occupation that would end up being life-long. Her main area of interest was astrology, which forced Viete to study the subject himself, effectively introducing him to the growing science world. During this time period, it is thought that Viete had already expressed $\sin mx$ and $\cos mx$ as polynomials in $\sin x$ and $\cos x$. [3]

When Catherine married four years later, the family moved to La Rochelle and soon after, Viete left for Paris in 1570. Upon becoming legal advisor to the Parliament de Paris, Viete was introduced to many prominent mathematicians. Four years later, he moved to Britanny to become an advisor to the Parlement seat at Rennes, and with a lighter workload, started to feed his growing mathematical interest. When Henri III came to power in 1574, Viete was recommended to him by various contacts and was soon being called upon for personal aid in confidential missions and negotiations [1]. His growing reputation led to his appointment as *maitre des requetes* at the court and a member of the privy council in 1580, bringing him back to Paris. Four years later, after being dismissed because of his relationship with the Huguenot families that caused problems with the "fervid and powerful Catholics", he returned to Fontenay. [3] In 1589 when Henri III moved the government to Tours from Paris, it was soon followed by Viete's return to the court.

This four-year break from courts proved to be one of the most creative and productive as he devoted his time to a vital work, *The Art of Analysis, or New Algebra*, that would unfortunately never be finished. [3] Upon reinstatement, Viete was to be the cryptanalyst for deciphering communication between those that opposed Henri. Spain used a cryptographic system involving more than 500 symbols to maintain secret communication to the colonies and with the Nederlands. [3] "He was so successful at this, we are told, that there were those, particularly in Rome, who denounced him by saying that the decipherment could only have been the product of sorcery and necromancy." [1] Even after Henri III's death a couple months later, Viete still remained a very influential figure in the court while purposely remaining in the background of the debates. In 1594, the court returned to Paris bringing Viete with it, being appointed as a privy councilor. Soon after, he was reappointed to a small mission in Poitou, which allowed him to live in his former home of Fontenay until his death in 1603.

Viete's life in law and politics is minute compared to his achievements in the sciences. Behind his duties in the court, Viete filled his mind with mathematics that would lead on to numerous texts where he claimed his fame in scientific history. "So profound was his meditation that he was often seen fixed in thought for three whole days in a row, seated at his lamp-lit dining room table, with neither food nor sleep except what he got resting on his elbow, and not stirring from his place to revivify himself from time to time." Now that his personal life has been investigated, the vast works that have been published can by analyzed so that the true impact of Viete's mathematics can be seen.

Of the lectures Viete gave to Catherine, only one has been found, appearing in a 1737 French translation. *Principes de comsographie*, illustrates Viete's "special interest in cosmological and astronomical questions….*All the mathematical investigations of Vieta are closely connected with his cosmological and astronomical work."* [2] Not only Viete shows this correlation, Kepler, Descartes, Barrow, Newton and others were very influential mathematicians whose interest in mathematics stemmed from cosmological interests. His main work, *Harmonium coeleste (Harmonic Construction of the Heavens)* was never published yet remained in manuscript until the nineteenth century. [2] "But the manner in which the founders of modern science set about attaining a mathematical comprehension of the world's structure betrays, from the onset, a different conception of the world, a different understanding of the world's being, than that which had belonged to the ancients….He wishes to be in every respect, the loyal preserver, rediscoverer and continuator of our ancient teachers. " [2] This revolutionary mentality that differed from the ancients is seen in all of Viete's work, as he transforms past mathematics into a new analytic science. For example, through his *Harmonium*, he renews Ptolemy's mathematical composition seen in his *Almagest,* while accepting the Copernican thesis, which is in accordance with the methods behind Ptolomy's science. [2] In a letter to Catherine he states the scope of his life's work:

> Those things, which are new, are wont in the beginning to be set forth rudely and formlessly and must then be polished and perfected in succeeding centuries. Behold, the art which I present is *new*, but in truth *so old*, so spoiled and defiled by the barbarians, that I considered it necessary, in order to introduce an entirely new form into it, to think out and publish a new vocabulary, having gotten rid of all its pseudo-technical terms, lest it should retain its filth and continue to stink in the old way…"[5]

In Paris during 1579, the royal printer Jean Mettarer published his *Canon Mathematicus* and the *Universalium Inspectionum Liber Singularis*, both of which were part of a series of four. [1,2] His *Canon* deals with the works of Regiomontanus and Rheticus, and was intended to be the trigonometric part to the mail work. The second work contains the computational methods used in the construction of the cannon (contructio Canonis) and teaches the computation of a plane and spherical triangles using the trigonometric relations between the "determining components" of each triangle. [2] The data that Viete found is presented in tables that describe the proportion (analogia) obtaining between three "known" and one unknown component of the triangle. This plays as an introduction to Viete emphasis on magnitudes and their ratios, which will be described later in his *Analytic Art*. His tables of trigonometric functions are incredible in depth: he computed these for every minute of arc to one part in 10,000,000. For example, is computation behind one minute of arc was based on an inscribed polygon of 6.144 sides and a circumscribed polygon of 12,288 sides, yielding a value of 29.083,819,59 on a base of 10,000,000. [1] Throughout the text, Viete gathered formulas for the solution of right and oblique plane triangles and adds his own contribution, the law of tangents [7]:

$$\frac{a-b}{a+b}=\frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

He went on to describe what equations to use such that any one part in terms of two other known parts for spherical right triangles, which later bcame known as Napier's rule. He also contributed the law of cosines involving the angles of an oblique spherical triangle as well as expanding upon trigonometric identities established by Ptolemy [7]:

$$\cos A = -\cos B \cos C + \sin B \cos C \cos a$$
$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

However, soon after it was released, Viete attempted to have The Op withdrawn because of errors, especially with spherical triangles and were reprinted in his *variorum de Rebus Mathematicis Responsorim Liber VII.* [1] These words directly coincide with the problems involving the formulation of equations seen in the contemporary algebraic work of Cardano, Tartaglia, Nonius, Bombelli, and Gossel. [2] These publications in 1579 close what Witmer refers to as Viete's first period of mathematic work.

However, before continuing, Viete's influences and contemporaries must be examined to properly gain the scope of his mathematical impact as he changed the methodology behind analysis. Work with equations and analysis far preceded his generation, and it can be assumed that these were a crucial part of his inspiration in the sciences. Viete puts considerable emphasis on the ancients. Viete's work draws from two important Greek sources, Pappus' seventh book, which played an essential role in the development of mathematics, and Diophantus' *Arithmetic*. The process that Viete builds upon known as analysis was first defined by Theon of Alexandria [2] for a process that is initiated by "the assumption of what is sought as though it were granted, and by means of the consequences [preceding to] a truth [which was in fact already] granted." [2] Viete attempts to recreate this ancient analysis and synthesis used by Greeks, which a shining example of is Diophantus' *Arithmetica*. At the beginning of each book in *Arithmetica*, Diophantus presents short algebraic introductions, and goes on to examine rational number, literal symbolism, and rules for polynomial and equation operations. He also introduced negative values to his concept of number, even though he did not set up rules for operations on them, and thus only used them as intermediate steps in solving for solutions in the positive rational domain. He is also credited with introducing literal signs for an unknown and its powers, which Viete definitely built upon to come up with his new version of analysis. [3] Like many other Renaissance mathematicians, Viete pays particular attention to Euclid's theory behind proportions, and used them as a method to solving equations. Viete used the ancient Greek methodology as a foundation for making advancements in how mathematics is performed.

During the twelfth century, Latin Europe gained knowledge of Arabic mathematics, including the decimal position systems as well as linear and quadratic equations through a translation of the work of Al-Khwarizmi. [5] He was active during the rule of the

Baghdad Caliph Al-Ma'mun (813-833) and was most likely part of the "House of Wisdom", which is described as a type of academy. His most notable works include *Arithmetic,* which introduced the Hindu-Arabic decimal position system, and *Algebra,* which contained much information about linear and quadratic equations. [5] He introduces the term of algebra, which was literally translated as being the transference of negative terms from one side of the equation to the other, and the combination of like terms on the or on the same side. [5] However, he lacks symbolic algebraic notation, and writes out each equation, only using specific numbers, as he organizes linear and quadratic equations into two groups, each containing three forms. Later, Nicolas Chuquet of Paris wrote an extensive work entitled *Le Tripart en la science de nombres du Maistre Nicolas Chuquet Parisien* (1484) where he approaches computation with rational and irrational numbers as well as the theory of equations. The third section became notable for its introduction of a new notation for exponential powers, replacing the "cossit" notation of special hieroglyphs. [5] Although the notation is not included in Viete's work, it is later seen in Descartes' work, which was most definitely impacted by the writings of Viete. Because of the depth that it contained, it showed that Lyons had considerable algebraic and arithmetic knowledge, comparable to the leading cities in Italy and Germany. [5]

The sixteenth century proved to be a huge period of advances in mathematics and the sciences, as the Renaissance was continually expanding upon the knowledge of the past. In 1545, physician, humanist, mathematician, and scientist-in-general Gerolamo Cardano produced a text (*Ars Magna)* that outlined the process of determining numerical solutions to third-degree equations and many quadratics and biquadratics, an essential step in the progress of algebra. [5] Nicolo Tartaglia 91499-1557) had entrusted Cardano with the method behind solving various cubic equations, and once Cardano broke this secrecy, the credit was given to him. "The *Ars Magna* was the best-known book on algebra, studied by all who were interested, and it lost this position only when Descartes introduced his new methods." [5] Cardano examines when imaginary numbers appear as roots to equations, and as a whole, excludes them from the *Ars Magna* with the exception of *casus irreducibilis*, where $x^3 = 15x + 4$ and $x = 4$ gives a real root that is the sum of the cube roots of two imaginaries, explained by the formula:

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}$$

Viete (1591) as well as Bombelli (1572) examine this example in later works. [5] The work concerning the biquadratics in the *Ars Magna* is due in part to Ludovico Ferrari, a young friend of Cardano that developed a method behind solving the equation: $x^4 + 6x^2 +36 = 60x$. "Though Viete is quite explicit about his indebtedness to the Greek writers on mathematics, particularly Diophantus, he leaves to surmise the answer to the question of how fully acquainted he was with the works of Cardano, Lodovico Ferrari, Nicolo Tartaglia, Simon Stevin and others of their time." [1] Viete provides limited citations to these works, and it is unlikely that he was unaware, as the translator of Ferrari's work into French lived in Paris while Viete was there. [1] However, the fact that Viete ignores many results of his contemporaries cause questions to arise concerning his familiarity of the others' work. "Whereas Cardano's algebraic novelty concerned new results and the expanding universe of algebraic objects, Viete's concerned the language and methods of algebra." [6] In general, these previous works in algebra and geometry effectively set the stage for Viete's work and genius, as his new notation and theology allowed for a progressive way of looking at problems and solving equations, building off the advancements made by those before him.

Viete's second "active mathematical period" began at around 1584, when he was dismissed from the court. It was during this five-year absence that Viete did the most work in mathematics, producing many works including his famous *Analytic Art (In artem analyticen isagoge)*(1591). He effectively reformulates the study of algebra by introducing the systematic and detailed study of equations in order to obtain their solutions. The text is divided up into various topics: Introduction to the Analytic Art. Preliminary Notes on Symbolic Logicistic, Five Books of Zetetica. Two Treatises on the Understanding and Amendment of Equations, On the Numerical Resolution of Powers by Exegetics, A Canonical Survey of Geometric Constructions, A Supplement to Geometry, and Universal Theorems on the Analysis of Angular Sections. Each of these sections are incredible works in their own aspect, and all contributed greatly to the advancement of mathematics.

The introduction to what Viete calls the analytic art, describes the new methods and notation that find their way into the rest of the text, as well as defining what separates his work from the ancients. The first chapter deals with the definition and partition of analysis, and on those things, which are of use to zetetics. First off, synthesis and analysis were defined prior to Viete by Pappus in his *Mathematical collection:*

> Analysis then takes that which I sought as if it were admitted and passes from it through its successive consequences to something which is admitted as the result of synthesis. Indeed, in analysis we assume that which is sought as if it were already done, and we inquire what it is from which this results…until we come upon something already known or belonging to the class of the first principles, and such a method we call analysis as being solution backwards. But in synthesis, reversing the process, we take as already done that which was last arrived at in the analysis, and, by arranging in their natural order as consequences what were before antecedents and successively connecting them one with another, we arrive finally the construction of what was sought; and this we call synthesis." [5]

Viete took this definition of synthesis and analysis and combined them with the constructions presented by Diophantus in order to form his own analytic form. Pappus was the one to introduce two types of analysis, "the one directed to the searching (*zetetikon)* for the truth and called theoretical, and the other directed to the finding (*poristikon)* of what we are told to find and called problematic" [5] In the zetetic approach, a proof is found that can be turned into a synthesis, and thus becoming the demonstration. In the poristic method, a solution is sought out that is then inverted to form a construction. The combination of these two is what Viete goes on to explain as being Zetetics:

> "art by which I found the equation or proportion between the magnitude that is being sought and the given things; a poristic art by which from the equation or proportion the truth of the required theorem is investigated, and an exegetic art by which from the construction equation or proportion there is produced the

> magnitude itself that is being sought….But what truly belongs to the zetetic art is established by the art of logic through syllogisms and enthymemes, of which the foundations are those very symbols by which equations and proportions are obtained…The zetetic art, however, has its own form of proceeding, since it applies its logic not to number--which was the boring habit of the ancient analysis but through a logistic which in a new way has to do with species. This logistic is much more successful and powerful than the numerical one for comparing magnitudes with one another in equations, once the law of homogeneity has been established and there has been constructed, for that purpose, a traditional series or scale of magnitudes ascending or descending by their own nature from genus to genus, by which scale the degrees and genera of magnitudes in equation s may be designated and distinguished.” [1]

With this new division and definition between analysis and synthesis, Viete effectively reforms the entire methodology behind algebra. Through this introduction, he explains the scope of his new form of analysis, and what is necessary in its application to problems. He reforms the analysis of the past to a broader sense, especially in application to equations, as he looks at the “species” of equations rather than the numbers contained in them. This proves to be an essential step in creating the proper analysis for the more advanced equations that he examines. Instead of finding roots based off specific coefficients, he introduces new notation that allows for the examination of equations based strictly off their structure and form.

In Chapter II of his introduction, Viete explains the importance of notation, as well as proportions. This definition allows for more concise and accurate explanation of equations as well as the methods behind finding their solution. Even though Viete is creating a completely new form of algebraic methods, he still incorporates various methods used in ancient mathematics. Struik points out various postulates that Viete used from Euclid:

1. The whole is equal to the sum of its parts.
2. Things that are equal to the same thing are equal among themselves.
3. If equals are added to equals, the sums are equal.
8. If like proportionals are added to like proportionals, then the sums are proportional. [5]

These postulates are just a few examples of how Viete is building off the mathematics of the past, and introducing new notation and methods in order to form an organized system behind analysis. He continues these definitions of his analysis in Chapter III, "On the Law of Homogeneous Quantities, and the Degrees and Genera of the Magnitudes that are Compared." "The first and supreme law of equations or proportions, which is called the of homogeneity, since it is concerned with homogeneous quantities, is as follows:

1. Homogeneous quantities must be compared to homogeneous quantities….
2. Magnitudes that ascend or descend proportionally in keeping with their nature from one kind to another are called scalar terms.
3. The first of the scalar magnitudes is the side or root….
4. The kinds of magnitudes of comparison, naming them in the same order as the scalar terms, are:

1. Length or breadth
2. Plane
3. Solid
4. Plano-plane

6.Solido-solid

9. Solido-solido-solid

5. In a series of scalar terms, the highest, counting up from the root, is called the power. The term of comparison [must be] consistent with this…

7. A supplemental term the product of which and a lower-order term is homogeneous with the power it [i.e.,the product] affects is called a coefficient.“ [1]

Through this chapter, Viete sets up the rules behind the operations that can be performed in an equation. Whether or not simple operations (addition or subtraction) can be performed depends on the homogeneity of the terms. The definitions of the various magnitude species is the basis for the power notation we see in the text, where he does not include the use of numbers as the exponents, and instead refers to them in term of their magnitude type (plano, solido) etc. Now that he has set up the rules for operations, as well as how he defines the types of terms and quantities through his analysis, he goes about how to define the equations in chapter IV, “On the Rules for the Calculation by Species[*logistica speciosa*].” “Numerical calulation [*logistica numerosa*] proceeds by means of numbers, reckoning by species by means of species or forms of things, as, for instance, the letters of the alphabet.” [1] Through this chapter, Viete effectively introduces the concepts behind analyzing equations based off their forms, rather than what the roots will end up being. Unlike those before him, he uses letters for coefficients so that more theory is put behind the operations performed. He lays out the ground-work for how operations will be used in solving equations, being sure to take note of the homogeneity throughout. The basic operations are examined in-depth, and are noted for how they change for what numerical values end up being introduced to the equation. Once of the most important operations found in Vietes analytic method of equations lies in his fourth rule, " To divide a magnitude by a magnitude.":

This leads in an analogous way to such expression as $\frac{Bplane}{A}, \frac{Bcube}{Aplane}$, and so forth. Furthermore, to add $\frac{Zplane}{G}$ to $\frac{Aplane}{B}$; the sum will be:

$$\frac{\text{G in A plane } + \text{ B in Z plane}}{B \ in \ G}$$

To multiply $\frac{Aplane}{B}$ by Z; the result will be $\frac{\text{A plane in Z}}{B}$.[5]

Even though he does not explain in this chapter, this is driving force behind his equation solving. The application of the dated method of proportions is widely used in Viete's construction and analysis. Instead of solving for a specific term, he sets up ratios on each side of the equation with a common term such that they can be compared. His geometric constructions are centered on finding the relation between two magnitudes, which is later algebraically analyzed. This proves to be yet another example of how Viete builds off the methodology of the past and applies it to new situations and equations in order to produce the most efficient process in analysis.

In the fifth chapter of the introduction, “Concerning the Laws of Zetetics,” Viete explains his new methodology behind analysis, and as a whole, completes the outline for the works that follow in *The Analytical Art*. He starts out by explaining the procedure behind finding the solutions to equations:

> -If we ask for a length, but the equation or poroportion is hidden under the cover of the data of the problem, let the unknown to be a found be a side.
>
> -If we ask for a plane…let the unknown to be found be a square.[5]

“These laws amount to introducing (1) x, (2) x^2, (3) x^3, (4) the law of homogeneity, as in $x = ab$; and to (5) denoting the unknown by vowels *A, E,…* and the given magnitudes by consonants, *B, G, D,…,* (6) construction $x^2 = ab + cd$, or, as Viete writes it: A square equal to *B* in *C* + *D* in *F*… [5]

This introduction to his most prolific work easily presents the bulk of his impact on mathematics. His new methods behind analysis and synthesis allowed for countless progressions following him, most notably new geometries as well as the founding of calculus. Obviously these advancements were most likely inevitable, but the fact that Viete applied many new theories to ancient mathematics is very noteworthy, a great representation of the theory behind the Renaissance in Europe. In the 1646 edition of the text, this paper was followed by *Ad logisticam speciosam notae priorae*, and in the original, it was followed by *Ad logisticam speciosame notae posteriores*, which is said to have been lost by the time the text was translate by Van Schooten. [5] *Notae priore* is a compilation of 56 propositions concerning algebraic identities and geometric problems, that continued on the work that Viete introduced. *Zeteticorum libri quinque* followed and was published in 1593 and was inspired by the work of Diophantus and presented more methods of equation solving by means of *logica speciosa*. Viete's contribution to mathematics can be easily presented by comparing methodologies in the following problem:

I. To divide a given number into two numbers with a given difference (Diophantus, *Arithmetic*, I, Prob. 1)

Diophantus: Let the given number be 100, and the difference be 40; let the smaller number be x, then the larger will be $x + 40$. Then $x + (x +40) = 2x + 40 = 100$, hence

$2x = 60$, $x = 30$, $x + 40 = 70$.

Viete: Let the given number be D, and the difference be B; let the smaller side be A, the the larger will be $A + B$. Then $A + (A + B) = A2 + B = D$, hence $A2 = D – B$, $A = D(1/2) –$ B(1/2), $A + B = D(1/2) + B(1/2)$.

In modern notation: $D = a$, $B = b$, $A =x$, then $x + (x + b) = a$, $2x = a – b$, $x = (1/2)(a – b)$, $x + b = (1/2)(a + b)$. [5]

The contribution to the evolution to what we know today as algebra becomes very clear when you can observe the progression that Viete inspired. His new concepts, notation, and methods behind analysis and synthesis helped shape math into what it has become.

A couple years following the completion of *The Analytic Art*, mathematician Adrian van Roomen (1561-1615) from the Netherlands published a treatise that contained the calculation of π to 17 decimal places and offered a challenge to other mathematicians to solve the equation: [3]

$x^{45} - 45x^{43} + 945x^{41} - \ldots - 3795x^3 + 45x = A$
where

$$A = \sqrt{1\frac{3}{4} - \sqrt{\frac{5}{16}} - \sqrt{1\frac{7}{8} - \sqrt{\frac{45}{64}}}}$$

The ambassador supposedly told Henry IV about the challenge, and added that no French mathematician was capable of solving the problem. Upon being informed, Viete immediately came up with a solution, and found 22 more negative solutions the next day. When Viete sent van Roomen his solution, he included a copy of his newly completed text, *Apollonius Gallus, or the Restored Geometry of "tangencies" by Apollonius of Perga*(1600), which impressed van Roomen so much that he came to Paris to befriend him. [3] This serves as the perfect segue to the following works of Viete concerning geometry. Since he was building off the synthesis and analysis of the past, geometry was an essential part to the work that was done. His new methods proved to be largely applicable to mathematically solve and explain the constructions of the ancient Greeks.

Shortly after *The Analytical Art*, Viete produced two works on geometry that built off the system he earlier established. In *A canonical survey of geometrical constructions*(1592) and *The Supplement of geometry*(1593), Viete examines the "exegetical" part geometric problem solving, constructions. [8] *The canonical survey* focuses on the applications of quadratic equations to geometrical problems, especially classical Euclidian constructions, whereas in *The Supplement*, he applies third and forth degree equations to constructions via his analytical method involving zetetics and poristics. As the analysis becomes more advanced algebraically, the constructions drift away from Euclidean geometry and the restrictions of strictly using straight lines and circles. This was essentially a starting point for the progression of analytic geometry, which would be built upon by mathematicians to follow. Years later, Descartes would expand upon this by introducing parabolic functions into constructions, as well as refining the advancements in notation initiated by Viete. In the *Isogoge* of 1591, Viete explains the importance of this supplement to classical geometry [8]:

> In order that, so to say, geometry itself supplies a deficient of geometry in the case of cubic and biquadratic equations, he [the learned analyst] assumes, when dealing with cubes and squared squares, that it is possible
>
> > *to draw, from any given point, a straight line intercepting any two given lines, the segment included between the two lines being prescribed beforehand, and possible.*
>
> This being conceded (it is, moreover, not a difficult assumption) famous problems that have heretofore been called irrational can be solved artfully: the mesographic problem, that of the trisection of an angle, finding the side of a heptagon, and all others that fall within those formulae for equations in which cubes, either pure of affected, are compared with solids and forth powers with plano-planes.

Bos goes on to explain that this assumption (neusis) was not new and would end up being the preferred method of Viete in constructions that went beyond using strictly the straight line and circle. “He showed that any geometrical problem leading to a third- or forth-degree equation could be reduced to either finding two mean proportionals between two given lines, or to trisecting a given angle.”[8] An application of neusis in Viete’s constructions included the trisection of an angle found in *The Supplement*(Prop 9, pp. 245-246 tr.) [8]

Given any angle ψ; it is required to find an angle φ equal to one third of ψ.

Construction:

1. Draw a circle with center *O* and radius *a*, and mark the horizontal diameter *AOB*; prolong *AB* to the left; take *C* on the circle such that $\angle COB$ is equal to the given angle ψ.

2. By neusis, draw *DEC* through *C* intersecting *AB* prolonged and the circle in *D* and *E*, respectively, such that $DE = a$.

3. Draw *EO.*

4. Then $\varphi = \angle EOA$ will be the required angle; that is $\angle EOA = 1/3\angle COB$.

[**Proof:** $DE = EO = OC$; the triangles *DEO* and *EOC* are isosceles; $\angle OEC = 2\varphi$, hence $\angle EOC = \pi - 4\varphi$, so $\angle COB = \pi - (\varphi + (\pi - 4\varphi)) = 3\varphi$.]

Instead of using neusis between straight line and circle, could have performed the trisection by neusis between straight lines as seen in Pappus' *Collection*.[8] Further in *the Supplement*, Viete explains the development of cubic and fourth-degree polynomials in problems involving synthesis and analysis, and their reduction in order to produce manageable constructions. In the same year as *The Supplement*, Viete produced another work entitled *Book VIII of various replies on mathematical matters* in which he continues his examination of constructions outside the realm of traditional methods involving the use of mean proportionals, the quadratix, as well as the spiral. He wrote on the use of spirals in squaring the circle: [8]

> Although the spirals are not described in the way of true knowledge, and neither are their tangents found in that way, still we can reason truly about questions of how large the angles are in the case of tangents, how large the lines are that are

> subtended by these angles, and thus art helps mechanics and mechanics helps art. This I wanted to show in this chapter, as well as a good method to square the circle as near to the true value as one wished; it is a not too difficult method and I don't think that a more general ant artful method can be proposed.

Like many mathematicians, Viete ventured into these infamous problems that were beyond the scope of classical methods. His new methods of algebra were applied to geometric problems, advancing constructions to upper-degree polynomials and their reduction. Yet again, Viete has used methods of the past in his geometrical and algebraic advancements that became vastly beneficial to the advancement of mathematics. Through most of his works he pushed the boundaries of algebra and its application to solving problems. His new definition of the processes of synthesis and analysis allowed for new precision and standardization in the approach to mathematical problems. As he introduced new notation and symbolic algebra, the form of equations could be examined and therefore properly applied to situations. With these advancements, new doors in mathematics were opened, allowing for its progression towards analytic geometry and calculus. Like many of the Renaissance scientist, Viete built off the discoveries and effectively modernized the classical methods behind both algebraic and geometric analysis. He created a new system for mathematical knowledge and will forever have a place in history as a founder of algebra as we know it today and in the quest to leave no problem unsolved.

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