

char1 (From Guenther and Lee *Partial Differential Equations of Mathematical Physics and Integral Equations* Section 2.1 number 6):

Suppose that $v > 0$ is a constant and that $f(t) \geq 0$ with $f(t) \not\equiv 0$ but with $f(0) = 0$. Solve

$$\begin{aligned} u_t + vu_x &= 0 && \text{in } x > 0, \quad t > 0, \\ u(x, 0) &= 0 && \text{in } x \geq 0 \\ u(0, t) &= f(t) && \text{for } t \geq 0. \end{aligned}$$

Interpret the solution graphically. Parametrize the initial curve in two parts.

char2 (From Guenther and Lee *Partial Differential Equations of Mathematical Physics and Integral Equations* Section 2.5 number 5):

This is a problem illustrating the local nature of the existence and uniqueness of the solution. Solve

$$\begin{aligned} u_t + uu_x &= 0, && -\infty < x < \infty, \quad t > 0, \\ u(x, 0) &= f(x), && -\infty < x < \infty. \end{aligned}$$

(a) Show that the solution is given implicitly by

$$u = f(x - tu)$$

If $f'(x) \geq 0$, show that there is a function $u = u(x, t)$ satisfying this implicit equation for u . If $f'(x) \leq -\alpha$, $\alpha > 0$ a constant, show that after a finite time the solution no longer exists.

(b) More explicitly suppose that

$$f(x) = \begin{cases} 1, & x \leq 0, \\ 1 - 3x^2 + 2x^3, & 0 < x < 1, \\ 0, & x \geq 1. \end{cases}$$

Take for the initial parameterization $x_0(\tau) = \tau$, $t_0(\tau) = 0$, $u_0(\tau) = f(\tau)$. Graph the characteristics for fixed values of τ , say τ equal to 0, $\frac{1}{2}$, 1, and interpret the results for $s > 0$. In certain physical applications it is nevertheless necessary to seek solutions for all time. Key words are shocks, discontinuous solutions, Rankine-Hugoniot relation (e.g. Guenther and Lee, Section 12-3).