

Review for Test 2

Remark: This sample test is too long and heavily emphasizes setting up integrals but not evaluating them. Be prepared to evaluate some integrals (this means being able to integrate using substitution and integration by parts).

1. Write an equation of the plane that passes through the point $(-1, 1, 2)$ and is parallel to the plane determined by the equation $6x + y + 3z = 26$.
2. Given the two planes $x + 2y + 3z = 6$ and $x - y + z = 0$ determine the angle between the two planes at the line of intersection.
3. Graph the plane $10x - 5y + 2z = 10$. Be sure to label your axis.
4. Sketch $z = x^2 + y^2$.
5. Calculate the gradient of $f(x, y, z) = \frac{x^2}{y} - z \cos(xz)$. Give a geometric interpretation.
6. You want to determine the volume, V of a rectangular box. You measure its dimensions to be $l = 4$ ft, $w = 3$ ft, $h = 2$ ft, but you suspect you might have a small mistake in one of the measurements, l , w , or h . Making a mistake of 0.2 in which measurement will cause (Hint: you may want to consider differentials)
 - (a) the least error, and
 - (b) the greatest error in determining V .
7. (8 pts) Use the chain rule to find $\frac{\partial w}{\partial r}$ when $r = 1$ and $\theta = 2\pi$ for the function given by $w = xy + yz$ where $x = r \cos(\theta)$, $y = r \sin(\theta)$, and $z = r^2$. (No points for not using the chain rule).
8. (8 pts) The surface of a mountain is described by

$$h(x, y) = 14,000 - .01x^2 - .02y^2$$

Suppose a mountain climber is at the point $(500, 300, 9700)$ (this is a Colorado mountain!).

- (a) In what direction should the climber move in order to ascend at the greatest rate?

- (b) Suppose it is now winter and a skier is at the same point, wanting to go downhill as fast as possible. Which direction should the skier ski in?

9. (8 pts) Find the equation for the plane tangent to

$$h(x, y) = e^x(\sin(y) + 1)$$

at the point $(0, \pi/2, 2)$. Simplify.

10. (8 points) Mark T for True or F for False (if true, must be true for *all* vectors \mathbf{u} and \mathbf{v} in three dimensions:

- (a) T F The following limit,

$$\lim_{(x,y) \rightarrow (0,0)} \frac{e^{xy}}{x^2 - \cos(y)}$$

exists.

- (b) T F The equation of the plane $2x + 3y - z = 2$ contains the point $(2, 1, 5)$.

11. Determine the location of the relative extrema for the following functions. For each extrema, determine whether it is a local max, local min, saddle point, or indeterminate. You do not need to find the z -value.

(a) (8 points) $f(x, y) = 2x^2 + y^2 + 12x - 4y + 20$

(b) (8 points) $f(x, y) = -x^3 + 4xy - 2y^2 + 1$

12. (8 points) Find the absolute maxima and minima of the function $f(x, y) = x^2 + 2y^2 - 2x + 3$ in the region $x^2 + y^2 \leq 4$.

13. Evaluate $\int_0^2 \int_{x/2}^1 x \, dy \, dx$.

14. Evaluate $\int_1^\infty \int_0^\infty e^{-2x-3y} \, dy \, dx$.

15. Set up the iterated integral needed to integrate $\frac{y}{1+x^2}$ over the region bounded by $y = 0$, $y = \sqrt{x}$, and $x = 4$. DO NOT EVALUATE.
16. Given $\int_0^1 \int_1^2 dx dy + \int_1^2 \int_y^2 dx dy$, change the order of integration to $dy dx$ and express it as a single iterated integral. DO NOT EVALUATE.
17. Set up an iterated integral to find the volume under the surface $z = 4 - x^2 - 2y^2$. DO NOT EVALUATE.

1) $6x + y + 3z = 1.$

2) $\cos(\theta) = \frac{2}{\sqrt{42}}$

3) Recall that you need three points to sketch a plane (just as you only need to graph two points to sketch a line). So we find the points where the plane intersects the three axis: Setting y and z to zero, we find that $x = 1$, and so $(1, 0, 0)$ is one point. Similarly we also find that $(0, -2, 0)$ and $(0, 0, 5)$ are also on the plane. Plot these three points and connect the lines.

4) For constant z values we have circles, so this object has cross-sections of circles parallel to the xy -plane. Looking at the trace in the yz -plane by setting $x = 0$ we see that we have a parabola. Thus we have circles whose radii grow as a parabola. Also note that z cannot take on any negative values. Thus we have a bowl that opens up, also known as an elliptic paraboloid.

5) $\nabla f = \left\langle \frac{2x}{y} + z^2 \sin(xz), -\frac{x^2}{y^2}, x \sin(xz) - \cos(xz) \right\rangle$ The vector normal to the surface

$f(x, y, z) = 0$ at $(x, y, z).$

6a) l , 6b) h

7) 0 8a) $-10\mathbf{i} - 12\mathbf{j}$ 8b) $10\mathbf{i} + 12\mathbf{j}$

9) $2x - z = -2$

10) T T

11a) $(-3, 2)$ local min 11b) $(0, 0)$ saddle point; $(4/3, 4/3)$ local max

12) Min.: $(1, 0, 2)$, Max: $(-1, \sqrt{3}, 12)$, $(-1, -\sqrt{3}, 12).$

13) $2/3$

14) $\frac{1}{6e^2}$

15) $\int_0^2 \int_{y^2}^4 \frac{y}{1+x^2} dx dy$

16) $\int_1^2 \int_0^x dy dx$

17) $\int_{-2}^2 \int_{-\sqrt{2-\frac{1}{2}x^2}}^{\sqrt{2-\frac{1}{2}x^2}} (4 - x^2 - 2y^2) dy dx$ (Remark: graph was provided on original exam)